Three-body problem: chaos

FizziQ activity · Middle school · 40-60 min

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With the Orbits and Gravitation simulation in FizziQ Web, run in a computer browser, middle school students build a three-sun system and explore deterministic chaos.

Learning objectives

- Configure a three-sun system in the FizziQ Web Orbits and Gravitation simulation
- Observe the chaotic evolution of the three trajectories under mutual gravitational attraction
- Discover sensitivity to initial conditions by modifying a parameter very slightly
- Identify the various possible behaviours (ejection, collision, capture, chaotic dance)
- Understand why the N-body problem has no general mathematical solution and requires numerical simulation

Instruments and sensors

FizziQ Web Orbits and Gravitation simulation

Materials

- Computer, tablet or smartphone with FizziQ Web
- FizziQ experiment notebook
- Note: the protocol remains adaptable to any comparable orbital/gravitational simulation tool.

Protocol

1. Open the Orbits and Gravitation simulation in FizziQ Web (Experiment → Simulations → Orbits and gravitation).
2. Set the distance scale to 1,000,000 km/pixel (ruler icon) and the time scale to 12 hours per frame (speed icon). Cancel any centering by clicking the X button in the Centering area to observe motion in a fixed reference frame.
3. Configure body 1 as the first Sun: select "Sun" on the mass slider (333,000 M⊕), speed 20 km/s, initial angle 0° (motion to the right). Drag it into the upper half of the window to leave room for the others below. Choose a yellow colour.
4. Add a new body via the "+" tab and configure it as the second Sun: mass 333,000 M⊕, speed 20 km/s, initial angle 180° (motion to the left). Drag it 80 million km below body 1, watching the "distances" panel in the upper right. Choose an orange colour.
5. Add a third body as the third Sun: mass 333,000 M⊕, speed 40 km/s, initial angle 0° (motion to the right). Drag it 240 million km below body 1 (i.e. 160 million km below body 2). Choose a red colour.
6. Verify the configuration: three vertically aligned Suns, distances 80 and 160 million km between successive pairs, opposite speeds between body 1 and body 2, doubled speed for body 3.
7. Click the green Start button to launch the simulation. Watch the coloured trajectories carefully: the three bodies attract each other, their trajectories tangle, some bodies are ejected far away, others approach dangerously.
8. Let the simulation run for several minutes (about 200 to 500 simulated days). Observe that no orbit closes regularly: the three bodies follow complex, unpredictable trajectories. Save a screenshot using the IMG button.
9. Test 2 — Sensitivity to initial conditions: stop the simulation, modify body 1's speed very slightly (from 20 km/s to 21 km/s, only a 5% variation). All other parameters remain the same. Restart the simulation and

observe that the trajectories obtained are completely different from the previous test.

10. Test 3 — Mass variation: replace body 1 with a "Red dwarf" (lower mass, around 100,000 M⊕). Restart the simulation and observe how the system evolves. Depending on the case, a body may be ejected to great distance or captured into orbit around another.

11. Test 4 — Distance variation: return to the initial configuration (3 Suns, 333,000 M⊕ each) but double the distance between bodies (160 and 480 million km). Restart and observe that interactions are slower and the system may seem more stable for the first dozens of days.

12. Test 5 — Adding bodies: return to the initial configuration and add a fourth body (e.g. an Earth-mass planet). Observe how this small body is tossed from one star to another without being able to settle into a stable orbit.

13. Fill in a 3-column summary table: Test, Modification made, Observed behaviour (collision, ejection, chaotic configuration, capture). Conclude that the three-body system displays a wide variety of unpredictable behaviours.

Expected results

With the initial configuration (three Suns vertically aligned, speeds 20, 20 and 40 km/s with opposite directions), the three bodies follow intertwined trajectories. Bodies 1 and 2 first approach each other, while body 3, faster, is gradually pulled towards the central pair. Depending on the moment, the three bodies may form a moving triangular configuration, eject each other or undergo a collision (the simulation then stops). No orbit closes properly as in a two-body system: the motion is chaotic. Modifying just one parameter slightly (speed +5%, different mass, doubled distance) yields a completely different evolution — this is the hallmark of deterministic chaos. Doubling the initial distances slows the interactions and the system may seem stable initially, but the chaotic character reappears in the long run. Adding a fourth body (planet) shows how it is tossed between the three stars without being able to settle into a stable orbit.

Questions

- Why is a three-body gravitational system fundamentally more complex than a two-body system?
- What does "sensitivity to initial conditions" mean and why does it make long-term prediction impossible?
- What happens when a body is ejected from the system? Where does its extra energy come from?
- Why does the FizziQ simulation use step-by-step calculation rather than a single formula?
- Could a habitable planet exist in a three-sun system? Why?

Going further

- Modify the speed of one sun very slightly (20 → 21 km/s for example) and compare the final system with the initial configuration to visualize sensitivity to initial conditions
- Give the three suns the same mass but different speeds (10, 20, 30 km/s) and observe whether some configurations produce more regular trajectories than others
- Configure three suns placed in an equilateral triangle with tangential speeds: this is a special configuration (Lagrange L4-L5 point) that can give a stable orbit known as the Lagrange configuration
- Add a small planet (Earth mass) near the three-sun trio and observe its extremely chaotic behaviour
- Reproduce the figure-eight orbit discovered in 1993 by mathematicians Cris Moore, then Alain Chenciner and Richard Montgomery: three bodies of equal mass following a single 8-shaped trajectory