Moment of inertia of a cylinder

FizziQ activity · High school · 30 min

Scan to open in FizziQ

Taped to a rolling cylinder, a phone measures linear and angular acceleration on an incline; FizziQ data lets students calculate the moment of inertia. High school.

Learning objectives

- Measure simultaneously the linear and angular acceleration of a rolling object
- Verify the rolling-without-slipping condition ($v = R\omega$)
- Calculate the moment of inertia from the measured acceleration
- Compare solid and hollow cylinders
- Understand the conversion between translational and rotational kinetic energy

Instruments and sensors

Angular velocity · Linear acceleration · Gyroscope · Accelerometer

Materials

- Smartphone or tablet with FizziQ
- A cylinder (bottle, roll, tube)
- An inclined plane with a groove or gutter
- Adhesive tape or rubber bands to fix the phone
- A protractor for the incline angle
- Note: the protocol remains adaptable to any comparable motion-sensor measurement tool.

Protocol

1. Build an inclined plane with a groove or gutter to guide the cylinder. An angle of 5° to 15° gives good results.
2. Firmly attach your smartphone to a cylinder (bottle, roll, tube) using adhesive tape or rubber bands. Ensure the phone's gyroscope axis is aligned with the cylinder's rotation axis.
3. Open FizziQ and select the Gyroscope (angular velocity) and Accelerometer (linear acceleration) instruments.
4. Place the cylinder at the top of the inclined plane. Start the recording.
5. Release the cylinder and let it roll freely to the bottom.
6. Stop the recording. Observe the angular velocity $\omega(t)$ and acceleration $a(t)$ graphs.
7. The angular velocity should increase linearly with time (constant angular acceleration): $\omega = \alpha t$.
8. Verify the rolling-without-slipping condition: $v = R\omega$, hence $a = R\alpha$, where R is the cylinder radius.
9. Calculate the moment of inertia I from the ratio between measured acceleration and $g \sin \theta$: $a = g \sin \theta / (1 + I/mR^2)$.
10. Compare the experimental moment of inertia with the theoretical value: $I = ½mR^2$ for a solid cylinder, $I = mR^2$ for a hollow cylinder.

Expected results

The angular velocity increases linearly with time, confirming constant angular acceleration. The measured acceleration for a solid cylinder is approximately 2/3 of $g \sin \theta$. The ratio $v/(R\omega)$ is close to 1.0, confirming rolling without slipping. The moment of inertia calculated from the acceleration agrees with the theoretical value within 10-20%.

Questions

- Why does a solid cylinder descend faster than a hollow cylinder of the same mass and radius?
- What happens if friction is insufficient to maintain rolling without slipping?
- How does the total kinetic energy split between translation and rotation for a solid cylinder?
- Why does the mass cancel from the acceleration formula?
- What would happen if you used a sphere instead of a cylinder?
- How could you make the experiment more precise?

Going further

- Race a solid cylinder against a hollow cylinder of the same radius: the solid always wins
- Compare with a sphere ($I = 2/5\ mR^2$) which descends faster than both cylinders
- Vary the incline angle and verify that the ratio $a/(g \sin \theta)$ remains constant
- Use the data to calculate the fraction of energy that goes into rotation
- Test whether the mass of the cylinder affects the acceleration (it should not)