Tree height with a theodolite

FizziQ activity · Middle school · 30 min

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Students aim the FizziQ smartphone theodolite at a treetop, measure the elevation angle and apply the tangent formula to calculate the height without climbing.

Learning objectives

- Measure the angle of elevation to the top of a tall object using the FizziQ theodolite
- Apply the tangent formula to calculate the height of an inaccessible object
- Account for the observer's eye height in the calculation
- Evaluate measurement uncertainty and identify sources of error
- Connect the experiment to the historical and practical applications of trigonometry

Instruments and sensors

Theodolite (inclinometer) · Accelerometer · Gyroscope

Materials

- Smartphone or tablet with the FizziQ app (Theodolite / inclinometer tool)
- A large tree or building
- A tape measure to measure horizontal distance
- Calculator
- Note: the protocol remains adaptable to any comparable angle-measurement tool.

Protocol

1. Choose a tall object to measure: a tree, flagpole, or building with a clearly visible top.
2. Stand at a point where you can see the top of the object clearly. Ensure there is a clear, level path between you and the base of the object.
3. Measure the horizontal distance from your position to the base of the object using a tape measure. Record this as d (in meters).
4. Open FizziQ and select the Theodolite tool (inclinometer mode).
5. Hold the smartphone at eye level and sight along the top edge toward the top of the object. When the crosshairs are aligned with the top, record the elevation angle α.
6. Measure your eye height h_eye (the height from the ground to your eyes) with a tape measure.
7. Calculate the height of the object: H = d × tan(α) + h_eye.
8. Repeat the measurement from a second distance (farther away or closer) and recalculate H. The two results should agree within measurement uncertainty.
9. Take the measurement from at least three different distances and calculate the average height and standard deviation.
10. If possible, verify your result by comparing with a known measurement (building plans, tree measurement by arborist, or shadow method).
11. Try measuring the angle from very close (steep angle) and very far (shallow angle). Discuss which distance gives the most accurate result.
12. Record all data in your FizziQ notebook: distance, angle, eye height, and calculated height for each trial.

Expected results

For a typical school tree of 10-15 meters, the measured height should be accurate to within ±1-2 meters (5-15% relative error). The main sources of error are the angle measurement (±1-2° with a smartphone, translating to ±0.5-1.5 m at typical distances), the distance measurement (±0.1 m with a tape measure), and the identification of the exact top of the tree (canopy versus trunk tip). Measurements from very close range (angle > 60°) tend to be less accurate because the tangent function amplifies angular errors at steep angles. Measurements from very far away give shallow angles where small angle errors translate to large height errors. An optimal measurement distance is roughly equal to the object's height (giving an angle near 45°).

Questions

- Why is the optimal measurement distance approximately equal to the height of the object?
- How does the uncertainty in the angle measurement affect the calculated height?
- What was Thales' shadow method, and how does it relate to the tangent formula?
- Could you use this technique to measure the depth of a canyon or the height of a cliff from below?
- How do modern surveying instruments achieve greater accuracy than a smartphone?
- What are three real-world applications of trigonometric height measurement?

Going further

- Use the shadow method as an alternative: measure the length of the tree's shadow and a reference stick's shadow, then calculate the height by proportion
- Measure the height of the same object from multiple distances and plot tan(α) versus 1/d to verify linearity
- Calculate the height of a building by counting floors and comparing with the trigonometric result
- Measure the angle of elevation to the top and the angle of depression to the base of an object on a slope
- Use triangulation (measuring angles from two positions a known distance apart) instead of measuring the distance directly