Pendulum: a = 2gh/r

FizziQ activity · High school · 40 min

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A smartphone swinging as a pendulum records centripetal acceleration with the FizziQ accelerometer; high school students verify a = 2gh/r and energy conservation.

Learning objectives

- Derive the theoretical relationship between release height and maximum centripetal acceleration for a pendulum
- Measure the centripetal acceleration at the lowest point of a pendulum swing using the smartphone accelerometer
- Verify the conservation of mechanical energy by comparing theoretical and measured accelerations
- Investigate how the release height affects the maximum acceleration
- Understand the connection between potential energy, kinetic energy, and centripetal acceleration

Instruments and sensors

Accelerometer (single-axis linear acceleration) · Accelerometer

Materials

- Smartphone with the FizziQ app (accelerometer)
- Strong string or wire to build a simple pendulum, at least 1 meter long
- Material for fixing the pendulum firmly at a pivot point
- Tape measure, to measure the pendulum length and the release heights
- Support for setting different release heights
- FizziQ experiment notebook, to record and plot the data
- Note: the protocol remains adaptable to any comparable acceleration measurement tool.

Protocol

1. Construct a simple pendulum by attaching the smartphone securely to a strong string or wire, at least 1 meter long. The phone should be firmly fixed so it does not swing independently of the string.
2. Measure the length of the pendulum L from the pivot point to the center of the smartphone. Record this value.
3. Open FizziQ and select the Accelerometer sensor. Choose the axis along the string direction (radial direction), which will measure the centripetal acceleration.
4. Before the first trial, calculate the theoretical prediction: for a release height h, the maximum centripetal acceleration at the bottom should be a_max = g + 2gh/L (including gravity).
5. Pull the pendulum to a small angle corresponding to a known height h_1 (measure h from the lowest point to the release position). Start with $h_1 \approx 5$ cm.
6. Start recording in FizziQ, then release the pendulum from rest. Let it swing for 3-4 complete oscillations.
7. Stop recording and identify the peak acceleration values on the graph. These peaks occur at the lowest point of each swing.
8. Record the average peak acceleration for the first 2-3 swings (before friction significantly reduces the amplitude).
9. Repeat with larger release heights: $h_2 \approx 10$ cm, $h_3 \approx 15$ cm, $h_4 \approx 20$ cm, $h_5 \approx 30$ cm.
10. For each height, record the measured peak acceleration and the theoretical prediction.

11. Plot a graph of measured peak acceleration versus release height h. It should be a straight line with slope 2g/L.
12. Compare the measured slope with the theoretical value 2g/L and calculate the percentage discrepancy.
13. Discuss whether the conservation of energy is confirmed by your data and identify the sources of any discrepancy.

Expected results

For a 1-meter pendulum, the theoretical centripetal acceleration at the lowest point (including gravity) increases linearly from about 10.8 m/s^2 (h = 5 cm) to about 15.7 m/s^2 (h = 30 cm). The measured values should follow this linear trend within 5-15% accuracy. The graph of peak acceleration versus h should be linear with a slope close to $2g/L \approx 19.6$ $m/s^2/m$ for a 1-meter pendulum. Energy losses from air resistance and string friction cause the measured acceleration to be slightly less than predicted, especially at larger amplitudes. The accelerometer itself may also show some noise and axis misalignment, contributing ±0.2-0.5 m/s^2 of uncertainty. Over multiple swings, the peak acceleration should decrease gradually as energy is dissipated.

Questions

- How does the principle of conservation of energy predict the centripetal acceleration at the lowest point?
- Why does the measured acceleration include a component of g even when the pendulum is at the lowest point?
- What happens to the energy that is lost during each swing? Where does it go?
- How does the pendulum length affect the relationship between height and acceleration?
- Why did Leibniz call kinetic energy vis viva (living force)?
- How could you modify this experiment to minimize energy losses?

Going further

- Vary the pendulum length and verify that the slope of the a versus h graph changes as 2g/L
- Record the acceleration over many swings and plot the peak acceleration versus swing number to quantify energy dissipation
- Compare the pendulum behavior with and without a heavy bob (add mass to the smartphone) to verify that the result is independent of mass
- Investigate whether the acceleration at the lowest point depends on the direction of swing (forward versus backward)
- Use the accelerometer data to reconstruct the velocity at the lowest point and independently verify energy conservation