Law of sines with a theodolite

FizziQ activity · High school · 40 min

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Students measure the angles of a triangle in the schoolyard with the FizziQ smartphone theodolite, then apply the law of sines to calculate unknown distances.

Learning objectives

- Measure angles between three fixed points using the FizziQ theodolite function
- Directly measure one side of the triangle and use the law of sines to calculate the others
- Verify the calculated distances by direct measurement when possible
- Understand the principle of triangulation and its historical applications
- Apply trigonometric relationships in a concrete, real-world context

Instruments and sensors

Theodolite · Magnetometer (compass) · Gyroscope

Materials

- Smartphone with the FizziQ application (theodolite function)
- An open space with three visible reference points
- A tape measure or a means of measuring distance
- Drawing materials (protractor, ruler, paper)
- FizziQ experiment notebook to record angles and results
- Note: the protocol remains adaptable to any comparable angle-measurement tool.

Protocol

1. Choose three clearly visible, fixed points in an open space (e.g., trees, posts, or buildings in a schoolyard). Label them A, B, and C.
2. Stand at point A and use the FizziQ Theodolite tool to measure the angle between the directions to points B and C. Record this angle as angle A.
3. Walk to point B and measure the angle between the directions to points A and C. Record this as angle B.
4. Walk to point C and measure the angle between the directions to points A and B. Record this as angle C.
5. Verify that the sum of the three angles is close to 180° (within ±5° is acceptable for smartphone measurements).
6. Using a tape measure, directly measure the length of one side of the triangle (e.g., side AB, the distance between points A and B). Record this as c (the side opposite angle C).
7. Apply the law of sines to calculate the other two sides: $a = c \times \sin(A) / \sin(C)$ and $b = c \times \sin(B) / \sin(C)$.
8. If accessible, directly measure sides a (BC) and b (AC) with the tape measure to verify your calculations.
9. Calculate the percentage error between the law-of-sines prediction and the direct measurement for each side.
10. Create a scale diagram of the triangle using a protractor and ruler. Verify that the diagram is geometrically consistent with your measurements.
11. Discuss the sources of error: angle measurement precision, alignment with the reference points, and the tape measure accuracy.
12. Try a second triangle with larger dimensions or with one side that is difficult to measure directly (e.g., across a pond or road), and use the law of sines to determine the inaccessible distance.

Expected results

The FizziQ theodolite typically measures angles with a precision of ±2-3°, which translates to approximately 5-10% uncertainty in the calculated side lengths. For a triangle with sides of 20-50 meters, the law-of-sines calculation should agree with direct tape measurements within 10-15%. The sum of measured angles should be close to 180°, with deviations indicating measurement errors. Larger triangles (with sides over 30 m) tend to give better results because the relative angle measurement error is smaller. Students should find that the accuracy of the result is most sensitive to the angle measurements, particularly when angles are close to 0° or 180° (where the sine function changes slowly).

Questions

- Why is it sufficient to know one side and two angles to determine the entire triangle?
- Under what conditions does the law of sines fail or give ambiguous results?
- How did surveyors use triangulation to map entire countries before GPS existed?
- Why is the accuracy of the angle measurements more critical than the accuracy of the side measurement?
- How would you use triangulation to measure the width of a river without crossing it?
- What is the difference between the law of sines and the law of cosines, and when would you use each?

Going further

- Use the law of sines to measure the height of a building or tree by creating a triangle with a known baseline and measured elevation angles
- Chain multiple triangles together (as in a real geodetic survey) to measure a long distance across a field
- Compare the precision of the FizziQ theodolite with a traditional compass and protractor
- Investigate the effect of triangle shape on accuracy: compare results for elongated versus equilateral triangles
- Use the law of cosines as an alternative method and compare the results with the law of sines calculation