Cycloid of a rolling wheel

FizziQ activity · High school · 40 min

Scan to open in FizziQ

Using FizziQ's kinematics module on smartphone or tablet, students track a point on a rolling bicycle wheel and compare the curve to the cycloid equations.

Learning objectives

- Track the trajectory of a point on the rim of a rolling wheel using video-based kinematic analysis
- Identify the resulting curve as a cycloid and describe its geometric properties
- Measure the period and amplitude of the cycloid and relate them to the wheel radius
- Understand the composition of translational and rotational motion that generates the cycloid
- Compare the experimental curve with the theoretical parametric equations $x = r(t - \sin t)$, $y = r(1 - \cos t)$

Instruments and sensors

Video (kinematics) analysis module · Camera

Materials

- Smartphone or tablet with the FizziQ application
- Video of a moving bicycle wheel, or the 'Cycloid' video from the FizziQ library
- Experiment notebook for analysis of results
- Note: the protocol remains adaptable to any comparable video analysis tool.

Protocol

1. Open FizziQ and navigate to the Kinematics module. Load the 'Cycloid' video from the FizziQ resource library, or use a video of a bicycle wheel with a visible marker on the rim.
2. Identify a reference scale in the video. The wheel diameter is ideal if known. Set the scale in FizziQ.
3. Set the origin of the coordinate system at the initial position of the marked point on the rim when it touches the ground.
4. Begin frame-by-frame tracking by clicking on the marked point on the rim in each successive frame.
5. Track through at least two complete revolutions of the wheel (the point should touch the ground twice after the starting position).
6. Aim for at least 20-30 data points per revolution for a smooth curve.
7. After completing the tracking, view the y vs. x position graph in the FizziQ notebook. You should see the characteristic arch-shaped cycloid curve.
8. Measure the horizontal distance between two consecutive cusps (points where the curve touches the x-axis). This distance should equal $2\pi r$, the circumference of the wheel.
9. Measure the maximum height of the curve. It should equal $2r$, twice the wheel radius.
10. Verify the relationship: horizontal period = π × maximum height (since $2\pi r = \pi \times 2r$).
11. Use the FizziQ tools to compare your experimental curve with the theoretical cycloid equations: $x = r(\theta - \sin \theta)$ and $y = r(1 - \cos \theta)$.
12. Note the cusp points where the rim point touches the ground: the velocity is momentarily zero at these points, which is visible as a clustering of data points.

Expected results

The position-position plot should show the characteristic arch shape of a cycloid, with smooth arches meeting the x-axis at sharp cusps. The horizontal distance between consecutive cusps should equal the wheel circumference (2πr), and the maximum height should equal the wheel diameter (2r). For a typical bicycle wheel with radius 33 cm, the arch height should be about 66 cm and the horizontal period about 207 cm. Data points will cluster near the cusps (where the point moves slowly) and be more widely spaced at the top of the arch (where the point moves fastest, at twice the wheel's translational speed). The experimental curve should match the theoretical cycloid well, with deviations mainly due to pointing precision and slight wheel wobble.

Questions

- Why does the marked point momentarily stop at each cusp of the cycloid?
- What is the speed of the marked point at the top of the arch relative to the ground? How does it compare to the speed of the wheel's axle?
- Why is the cycloid called the brachistochrone curve? What does this mean physically?
- How would the curve change if the marked point were located halfway between the rim and the axle instead of on the rim?
- What is the total length of one arch of a cycloid? How does it compare to the circumference of the wheel?
- How did the cycloid contribute to the development of calculus in the 17th century?

Going further

- Track a point at different radial distances from the center (on the rim, at half-radius, at the hub) and compare the resulting curves (cycloid, curtate cycloid, prolate cycloid)
- Investigate the tautochrone property by analyzing the time it takes a ball to roll to the bottom of a cycloidal track from different starting heights
- Record your own video with a reflective marker attached to a bicycle wheel and analyze the resulting cycloid
- Compare the cycloid with a simple sine wave to highlight the differences in curvature near the cusps
- Calculate the instantaneous speed of the rim point at different phases and verify that it ranges from 0 (at the cusp) to 2v (at the top), where v is the wheel speed