Centripetal acceleration: $a = \omega^2 R$

FizziQ activity · High school · 30 min

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Spinning with a phone at arm's length, high school students record acceleration with FizziQ's accelerometer, derive ω from timed rotations and verify $a = \omega^2 R$.

Learning objectives

- Measure centripetal acceleration using the smartphone's accelerometer.
- Calculate angular velocity ω from the number of rotations and measured time.
- Experimentally verify the relationship $a = \omega^2 \cdot R$.
- Analyze sources of discrepancy between theoretical and experimental values.
- Interpret an acceleration vs. time graph.

Instruments and sensors

Single-axis accelerometer (X-axis acceleration measurement) · Accelerometer

Materials

- Smartphone or tablet with the FizziQ app
- Measuring tape or ruler to measure arm length
- Clear space to spin freely without obstacles
- FizziQ experiment notebook to record and analyze the acceleration data
- Note: the protocol remains adaptable to any comparable accelerometer-based measurement tool.

Protocol

1. Open the FizziQ app on your smartphone.
2. In the Measurements menu, select the accelerometer and choose the X Acceleration component (horizontal axis when smartphone is held vertically).
3. Measure your extended arm length from your rotation axis (your body) to the center of the smartphone. Note this value R in meters (typically between 0.60 m and 0.80 m).
4. Hold the smartphone vertically, screen facing you, arm extended horizontally. The smartphone's X axis is then directed along the radius of the circle you will describe.
5. Press the FizziQ record button to start data acquisition.
6. Rotate steadily on yourself while keeping your arm well extended and horizontal. Complete exactly 3 full rotations at constant speed.
7. Stop recording once the 3 rotations are complete.
8. On the acceleration graph in your notebook, note the total rotation time t in seconds. Calculate the period $T = t / 3$, then the angular velocity $\omega = 2\pi / T$ (in rad/s).
9. Calculate the theoretical centripetal acceleration: $a_{th} = \omega^2 \times R$.
10. In FizziQ, display the X acceleration graph vs. time. Identify the zone corresponding to regular rotation (excluding start and stop phases).
11. The average is displayed by pressing the Statistics button.
12. Compare the experimental average acceleration value with the theoretical value $a_{th} = \omega^2 \times R$. Calculate the relative error between the two values.

Expected results

The measured acceleration along the X axis shows a relatively constant value during the regular rotation phase, framed by transitional phases at start and stop.

For a rotation period of 2 s and an arm of 0.70 m, the theoretical centripetal acceleration is approximately 6.9 m/s². The average value measured by the accelerometer should be of the same order of magnitude.

Fluctuations around the average value are visible on the graph, due to irregularities in rotation and parasitic movements of the arm and hand.

The relative error between theoretical and experimental values is typically between 5% and 20%, depending on rotation regularity and timing precision.

Questions

- What happens if you double the rotation speed? Is the acceleration doubled or quadrupled?
- How does centripetal acceleration vary if you shorten your arm (smartphone closer to body)?
- Why doesn't the accelerometer measure the gravitational component on the X axis when the smartphone is vertical?
- What are the main sources of error in this experiment and how can they be reduced?
- What is the relationship between centripetal acceleration felt on a merry-go-round and that measured here?

Going further

- Vary the rotation speed (spin faster or slower) and plot a vs. ω^2 to verify proportionality.
- Modify the radius by holding the smartphone at different distances from your body (bent arm, extended arm) and compare accelerations.
- Compare results obtained by several students with different arm lengths.
- Use the FizziQ Web Centrifuge simulation to compare experimental results with simulation.